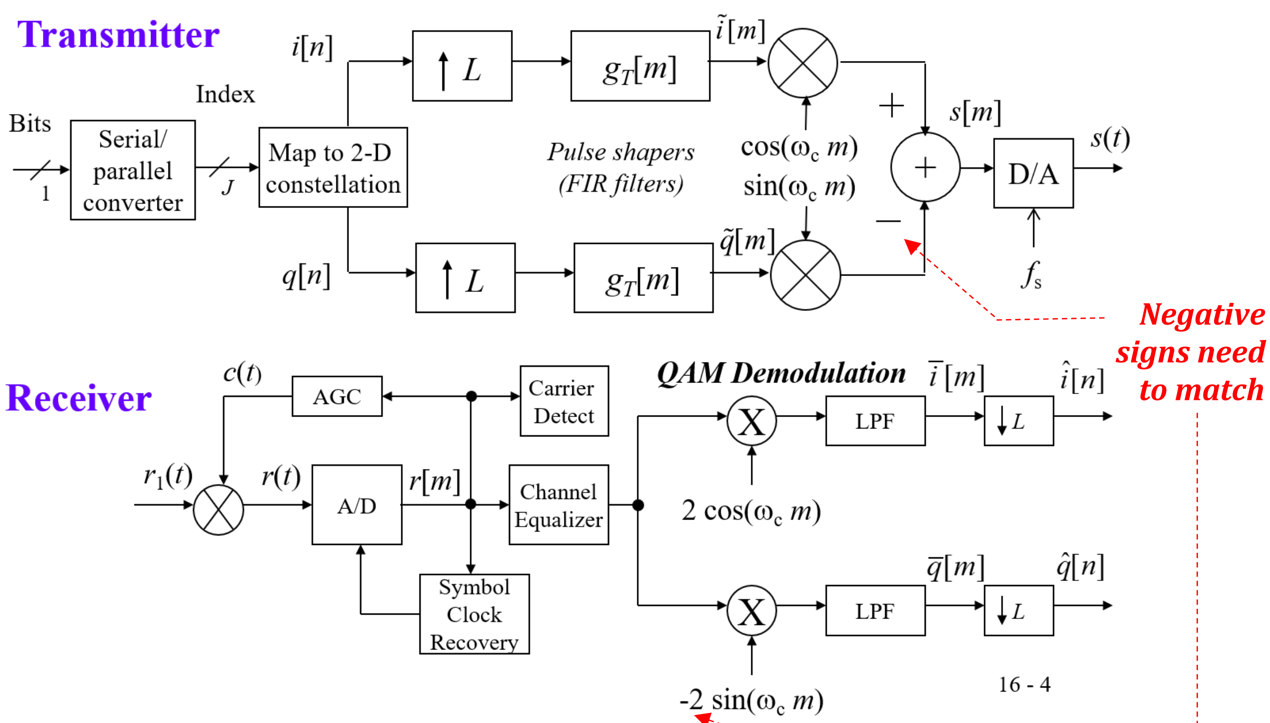
[10:40] QAM Receivers (Lecture Slides 16-2 to 16-4)

- Receiver must compensate for channel impairments and impairments in analog/RF processing in transmitter and receiver before baseband QAM demodulation

Impairment	Receiver subsystem
Fading	Automatic gain control
Additive noise	Matched filter
Linear distortion	Channel equalizer
Carrier mismatch	Carrier recovery
Symbol timing mismatch	Symbol clock recovery

- When transmitting is not transmitting, receiver can measure noise and interference
- Remove additive narrowband interference in transmission band using notch filter

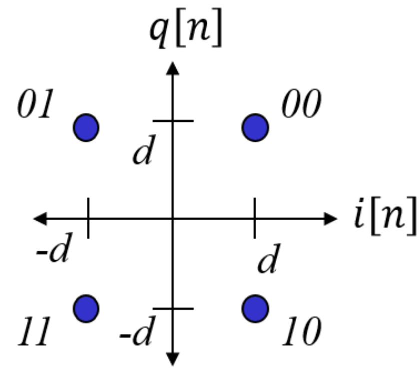


Matched Filter (Lecture Slides 16-3 and 16-4)

- Attenuates (filters out) noise outside the baseband bandwidth to increase SNR
- Maximizes SNR when impulse response $h_{opt}(t) = k\, g^*(T_{sym} - t)$ for real constant k
  - Transmitter pulse shape $g(t)$ is even symmetric, so $g(-t) = g(t)$
  - $g(t)$ is real, so the complex conjugation has no effect
  - Delay flipped $k\, g^*(T_{sym} - t)$ to make it causal— a delay by an integer multiple of T_{sym} will also maximize SNR per Lecture 14— and hence $h_{opt}(t) = g(t)$
- Discrete time: $h_{opt}[m] = k\, g^*(L\, T_s - m\, T_s) = k\, g^*[L - m] = k\, g[m] = g[m]$

Constellation Map (Lecture Slide 16-4)

- Grey coding
  - Adjacent symbols have one bit difference
  - Minimizes number of bit errors when there is a symbol error
- Error correcting codes
  - Send redundant bits to detect and correct errors
  - Reduces data rate
  - Reduces need for retransmission

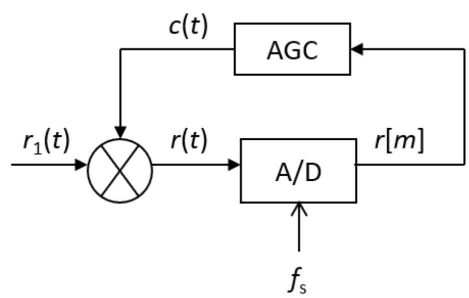


[11:15] Automatic gain control (Lecture Slide 16-5)

- Need to match the received voltage to the range of the A/D converter
- Increase or decrease gain if observed $r(t)$ is too high or low

Example: 8-bit signed A/D converter example

- When $c(t)$ is zero, A/D output is 0
- When $c(t)$ is ∞, A/D output is -128 or 127
- Approach: count number of values indicating gain is too high or low, and adjust gain
- $f_i = c_i/N$
- $c_i =$ count of times i occurs in N samples



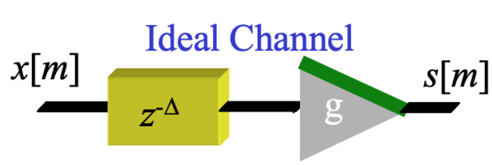
$$c(t) = \left(1 \quad \underbrace{+2 f_0}_{\substack{\text{increase gain} \\ \text{when 0 observed}}} \quad \underbrace{- f_{-128} - f_{127}}_{\substack{\text{decrease gain when} \\ \text{-128 or 127 observed}}} \right) c(t - \tau)$$

[11:35] Channel equalizer (Lecture Slides 16-6 to 16-8)

- Mitigates linear distortion in channel
- Time-domain perspective: shortens channel impulse response
- Frequency domain perspective: compensate for frequency distortion

Ideal channel

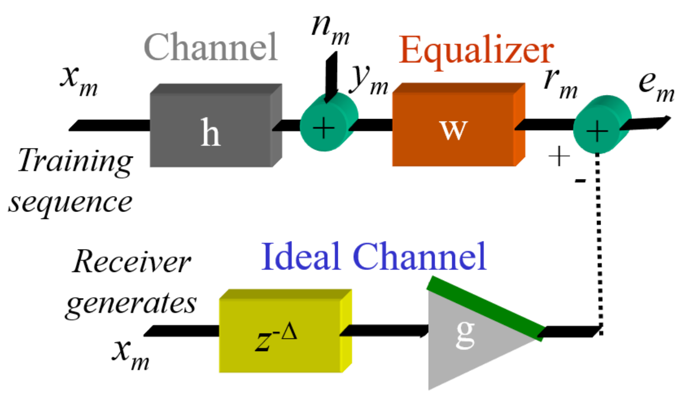
- Cascade of delay Δ and gain g
- Impulse response $g\,\delta[n - \Delta]$
- Frequency response $g\, e^{-j\,\Delta\,\omega}$
- Recover $x[m]$ from $s[m]$ by
  - Discarding first Δ samples
  - Scaling by $1/g$



See marker board notes for Lecture Slide 5-13

Adaptive FIR Equalizer

- Error $e[m]$ is difference between what we have $r[m]$ & what we'd like to have $s[m]$
- Adapt equalizer coefficients using gradient descent
  $$e[m] = r[m] - s[m]$$
  $$s[m] = g\, x[m - \Delta]$$
- Adaptive two-coefficient FIR equalizer: $w_0 = 1$ and adapt w_1
  $$r[m] = y[m] + w_1 y[m-1]$$



- Every time the receiver receives a sample, the update equation is calculated.
- Least mean squares objective: $J_{LMS}[m] = \frac{1}{2} e^2[m]$
- By driving $e^2[m]$ to zero, we drive the error $e[m]$ to zero.
- Update equation for w_1 to minimize $J_{LMS}[m]$:

$$w_1[m+1] = w_1[m] - \mu \left. \frac{\partial J_{LMS}[m]}{\partial w_1} \right|_{w_1 = w_1[m]}$$

$$w_1[m+1] = w_1[m] - \mu \underbrace{e[m]}_{\frac{\partial J}{\partial e}} \underbrace{y[m-1]}_{\frac{\partial e}{\partial w_1}}$$

- Similar update equation for other coefficients. For coefficient K,

$$w_K[m+1] = w_K[m] - \mu \underbrace{e[m]}_{\frac{\partial J}{\partial e}} \underbrace{y[m-K]}_{\frac{\partial e}{\partial w_K}}$$

- We can vectorize the update equation for a vector of N FIR coefficients

$$\vec{w}[m+1] = \vec{w}[m] - \mu\, e[m]\, \vec{y}[m]$$

where $\vec{w}[m] = [\, w_0[m] \quad w_1[m] \quad \ldots \quad w_{N-1}[m] \,]$
$\vec{y}[m] = [\, y[m] \quad y[m-1] \quad \ldots \quad y[m-(N-1)] \,]$

Run-time complexity of $2N + 2$ multiplication operations/sample (vs. N for FIR filter)

IIR Equalizer

- Mentioned in the discussion of Lecture Slide 5-3 on Channel Equalization
- We would like the cascade of the channel h and equalizer w to be an ideal channel:

$$H(z)\, W(z) = g\, z^{-\Delta} \text{ which means } W(z) = \frac{g\, z^{-\Delta}}{H(z)}$$

- Assume that $H(z)$ is FIR. Zeros of $H(z)$ become the poles of $W(z)$.
- We discussed adjusting poles of $W(z)$ to guarantee BIBO stability when discussing the solution to Fall 2019 Midterm #1 Problem 2. A predistorter would occur before the channel, and a channel equalizer would be after the channel. Same design approach.